**A STUDY OF HUMAN POPULATION GROWTH**

**(2015-2080)**

By

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**STEP ONE: INTRODUCTION AND ASK THE QUESTION**

As we look back through time, there have been many species that have lived on Earth. We can consider many species, such as bacteria, viruses, animals, and even mankind. Basic logic would dictate that the species at the top of the “food chain” would take over the world and become the sole inhabitant. In the most domains, human beings do currently sit atop that so called order. However, sole habitation by mankind has not occurred.

You may ask yourself, “why hasn’t mankind completely dominated all other life forms and taken sole habitation of the planet”? In all truth, most of the reason lies in the fact that, like many other species, human beings have a carrying (limiting) capacity that will not allow for exponential growth. In this study, I will examine various mathematical models and the variables that impact those models. I will also use those models and compare them to historical data to predict future values. Finally, I will select which method is the most accurate and useful to use based on agreement with the data.

Mathematical modeling is one of the best ways to approach studies/questions such as these. Math modeling allows us to use mathematical figures to describe behaviors in the real world and then use that knowledge to predict future activities with more accuracy. In this example, the question I pose is very simple. When we consider the human population growth over the 65 year period of 1950-2015, can we determine a viable mathematical model to determine what the population will be in another 65 years after that (the year 2080). Or said another way, based on what we know, “What do we predict the human population on Earth to be in the year 2080.

*Variables and Assumptions*  
 As I begin to examine this question, I must determine what variables I need to consider and what assumptions (if any) that I will make. Below is a list of those variables.

Po = Initial Population = 2,536,275,000

PN = New Population

R = Growth Rate = 1.6574% Annually

T = Time (In Years) = 65 Years

K = Carrying (Limiting) Capacity = 10,000,000,000

The assumption values being used for the variables are based on what I have uncovered through research of data that is already known. First we will take a look at the initial population of human beings on Earth for our study, in this case the value taken from World Census data in 1950. Based on the table below (Table 1), it is evident that value is 2,536,275,000 people.

Next, I had to calculate the average growth rate to use in my models. To calculate the average growth rate, I took a final population divided by an initial population and raised that to the power of the inverse of the time period. For the UN Data, that was the 2015 population divided by the 1950 population then raised to the power of 1/65 since it was a 65 year time frame. Once I got that value, I subtracted 1 and multiplied by 100 to get the 1.6574% figure stated above. The equation (with values) was:

(((7383009000/2536275000)^(1/65))-1)\*100

The final variable that needs explained that I looked at is carrying capacity. I will go into further depth on this topic later, but wanted to discuss for a moment now. As we all know, human beings require several things to survive. To name a few, our species needs food, water, and space to live. Because of these constraints, author Natalie Wolchover found in her research and published in her article *How Many People Can Earth Support* that “Many scientists think Earth has a maximum carrying capacity of 9 billion to 10 billion people”. For the purpose of this study, I will use the upper limit of that estimate so we can see what an optimal population could be and not limit ourselves further.

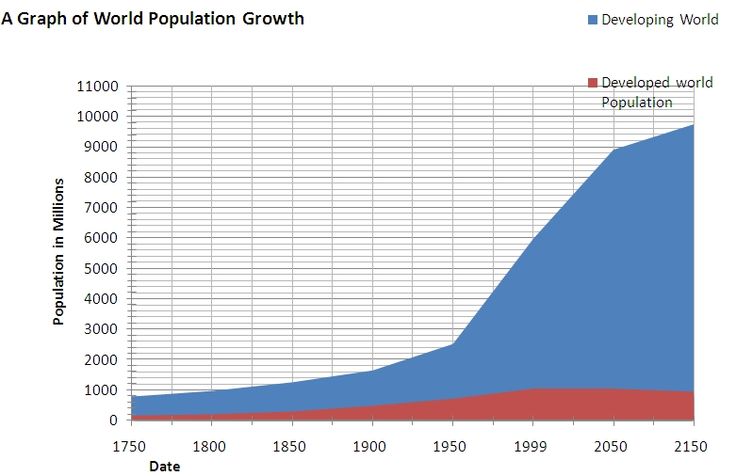
Table 1 – UN Human Population Data 1950-2015

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **TOTAL WORLD HUMAN POPULATION (IN THOUSANDS) BY YEAR 1950-2015** | | | | | | | |
| YEAR | POPULATION | YEAR | POPULATION | YEAR | POPULATION | YEAR | POPULATION |
| 1950 | 2 536 275 | 1967 | 3 479 054 | 1984 | 4 786 484 | 2001 | 6 223 412 |
| 1951 | 2 583 817 | 1968 | 3 551 881 | 1985 | 4 873 782 | 2002 | 6 302 150 |
| 1952 | 2 630 584 | 1969 | 3 625 906 | 1986 | 4 963 633 | 2003 | 6 381 409 |
| 1953 | 2 677 230 | 1970 | 3 700 578 | 1987 | 5 055 636 | 2004 | 6 461 371 |
| 1954 | 2 724 302 | 1971 | 3 775 791 | 1988 | 5 148 557 | 2005 | 6 542 159 |
| 1955 | 2 772 243 | 1972 | 3 851 545 | 1989 | 5 240 735 | 2006 | 6 623 848 |
| 1956 | 2 821 383 | 1973 | 3 927 539 | 1990 | 5 330 943 | 2007 | 6 706 419 |
| 1957 | 2 871 952 | 1974 | 4 003 448 | 1991 | 5 418 759 | 2008 | 6 789 771 |
| 1958 | 2 924 081 | 1975 | 4 079 087 | 1992 | 5 504 401 | 2009 | 6 873 741 |
| 1959 | 2 977 825 | 1976 | 4 154 288 | 1993 | 5 588 095 | 2010 | 6 958 169 |
| 1960 | 3 033 213 | 1977 | 4 229 201 | 1994 | 5 670 320 | 2011 | 7 043 009 |
| 1961 | 3 090 305 | 1978 | 4 304 377 | 1995 | 5 751 474 | 2012 | 7 128 177 |
| 1962 | 3 149 244 | 1979 | 4 380 586 | 1996 | 5 831 565 | 2013 | 7 213 426 |
| 1963 | 3 210 271 | 1980 | 4 458 412 | 1997 | 5 910 566 | 2014 | 7 298 453 |
| 1964 | 3 273 671 | 1981 | 4 537 846 | 1998 | 5 988 846 | 2015 | 7 383 009 |
| 1965 | 3 339 593 | 1982 | 4 618 776 | 1999 | 6 066 867 |  |  |
| 1966 | 3 408 121 | 1983 | 4 701 531 | 2000 | 6 145 007 |  |  |
|  |  |  |  |  |  |  |  |

Graph 1 – Human Population 1950-2015

\*Note – The dots on the graph represent the actual population values. I included an exponential curve example to show how historical data has actually acted. As you can see the actual data “weaves” around the curve in an “s-shape”. This observation will be explored in subsequent sections.

Figure 1 – Historical World Population Example (1750-2150 Projected)



\*Note: Again, notice the “s-shaped” nature of the blue curve.

**STEP TWO: SELECT MODEL TYPE**

For the models that I will formulate, I will be taking a dynamic model approach. Dynamic math models are used in cases where behaviors of the equations change over a period of time. These models allow us to look at various behaviors of differing time periods of the model, so this would be the most applicable approach in this case. Mostly I will be focusing on the use of continuous time models. Again, this allows us to examine situations that have a constant flow of time and we can then analyze on any period we wish.

**STEP THREE: FORMULATE THE MODELS**

In my study, I want to examine two popular types of growth models. I will build a model with no limitations (an exponential growth model-fixed percentage approach) and one with a carrying capacity (logistical growth model). I will then compare both models and their projections to historical data of human population growth and choose the model I feel is more appropriate to use going forward to project future values.

*Exponential Growth Model*

First, I will formulate an exponential growth model. We have an initial value of the population which is 7,383,009,000 people. We also know the average annual rate of growth based on the UN population census data from 1950-2105 (1.6574%). We don’t have to account for any other mitigating factors as the growth rate already takes into account all positive and negative impact variables due to the properties of averages. Some of these variables would include natural death rate, reproduction deficiencies, reproduction boom (i.e. after World War II and the baby boomers), etc.

Here are the factors that are known:

Po = Initial Population = 7,383,009,000 human beings

G = Growth Rate = 1.6574% increase =1.016574 annually

T = Time in Years = 65

Pn = New Population

So therefore, I am ultimately trying to determine a new population figure at a particular time in the future based on determining an average growth rate from past data and an initial population and extrapolating that information. This situation calls for a recursive formula, where each year builds upon the numerical answer from the prior year. If we take the growth rate and raise it to the power of the year we are looking for, and then we multiply that figure by the initial population, we can determine the new population for any given year in the future. Mathematically, this relationship is represented by the following formula:

Pn = Po x GT

*Logistical Growth Model (Carrying Capacity)*

I am trying to determine what the human population will be on earth in the year 2080 (65 years from now). I am looking to determine the total number given that the human species does not grow exponentially, but rather has a limiting (or carrying) capacity that will not allow the species to go beyond a certain number humans on earth. This carrying capacity accounts for all impacting variables on population (food and water source limitations, space limitations, diseases due to population density, etc.).

Here, I am accounting for many variables and making several assumptions. First, as in the exponential model, I am assuming that the initial population is 7,383,009,000 people. Let’s say that this is the first discovery that humans actually populate Earth by an outside source and the outside source states that this is the case based on their observations.

I am also assuming the carrying capacity to be 10 billion human beings.

We must take into account the following variables:

P0= Initial Population = 7383009000 people

P(t) = Population at time t

T = Time = 65 years

K = Carrying Capacity = 10 billion people

r = Growth Rate = 1.6574%

To account for all the variables needed when calculating a carrying capacity model, we must remember what we are trying to accomplish with this question. We are looking at the change in population over a set time frame. Therefore, we must involve differential equations as we have to consider a change in one variable with respect to another. In this case, the change we are looking at is the change in population with respect to time.

We start with the equation:

dP/dt = rP (1-K/P)

Again, we must eventually get to a point where we can consider population as a function of time so we can determine an actual numerical solution at any given time.

We must separate the variables and integrate both sides. We also must substitute for some variables during our integration process.

∫dP/P + ∫dP/K-P = ∫r dt

ln |P| - ln |K-P| = kt+C

ln |(K-P)/P| =-kt-C

Furthering the model, we get

|K-P|/P = e-rT-C

(K-P)/P = (A)e-rT-C Where (A=K-P0/P0)

Ultimately, we get the following equation of

P(t) = (K)/(1+((K-P0)/P0)\*e-(rt)))

**STEP FOUR: SOLVE THE MODEL(S)**

*Exponential Growth Model*

The model is fairly straight forward and simple to solve. The mathematics show that if you begin with the initial population of 7383009000 and multiply by 1.016574^(t) you get the new population for any value of t chosen. The new population is the initial population for the next year. The data table below shows those figures for all years from 2015-2080.

Table 2: Projected human population by year 2015-2080 – Exponential Growth Model

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **EXPONENTIAL GROWTH EXAMPLE OF HUMAN POPULATION (IN THOUSANDS) ON EARTH (YEARS 2015-2080)** | | | | | | | |
| YEAR | POPULATION | YEAR | POPULATION | YEAR | POPULATION | YEAR | POPULATION |
| 2015 | 7383009 | 2032 | 9763294 | 2049 | 12910983 | 2066 | 17073489 |
| 2016 | 7505375 | 2033 | 9925110 | 2050 | 13124970 | 2067 | 17356465 |
| 2017 | 7629769 | 2034 | 10089609 | 2051 | 13342503 | 2068 | 17644131 |
| 2018 | 7756225 | 2035 | 10256834 | 2052 | 13563642 | 2069 | 17936565 |
| 2019 | 7884776 | 2036 | 10426831 | 2053 | 13788446 | 2070 | 18233846 |
| 2020 | 8015459 | 2037 | 10599646 | 2054 | 14016975 | 2071 | 18536053 |
| 2021 | 8148307 | 2038 | 10775324 | 2055 | 14249293 | 2072 | 18843270 |
| 2022 | 8283357 | 2039 | 10953914 | 2056 | 14485460 | 2073 | 19155578 |
| 2023 | 8420645 | 2040 | 11135464 | 2057 | 14725542 | 2074 | 19473063 |
| 2024 | 8560209 | 2041 | 11320024 | 2058 | 14969604 | 2075 | 19795809 |
| 2025 | 8702086 | 2042 | 11507642 | 2059 | 15217710 | 2076 | 20123905 |
| 2026 | 8846314 | 2043 | 11698369 | 2060 | 15469928 | 2077 | 20457439 |
| 2027 | 8992933 | 2044 | 11892258 | 2061 | 15726327 | 2078 | 20796500 |
| 2028 | 9141982 | 2045 | 12089360 | 2062 | 15986975 | 2079 | 21141182 |
| 2029 | 9293501 | 2046 | 12289729 | 2063 | 16251943 | 2080 | 21491576 |
| 2030 | 9447532 | 2047 | 12493419 | 2064 | 16521303 |  |  |
| 2031 | 9604115 | 2048 | 12700485 | 2065 | 16795127 |  |  |

As you can see from the graph, this leads to an exponential growth curve that is j-shaped, or will continue to ascend at a steeper degree with each passing year. The graphs of the years 2015-2080 (new projection) and 1950-2080 (historical data plus projection) are shown below.

Graph 2: Exponential growth model projection human population 2015-2080

Graph 3: Exponential growth model projection human population 1950-2080

*Logistical Growth Model (Carrying Capacity)*

Below is the table of projected values by year from 2015-2080 based on the model built for logistical growth. I have also included a graph of the projected growth figures from 2015-2080 and a graph of the combined historical data along with the projected data. As you can see, it does have the tell-tale “s-curve” shape that occurs when there is a limiting capacity.

Now that we have the model built, the following shows the computation of the population after 65 years.

P(65) = 10000000000/(1+(((10000000000-7383009000)/ 7383009000)\*e-((.016574)(65))))

P(65) = 10000000000/(1+(.35446)(.34051))

P(65) = 10000000000/1.120697

P(65) = 8923012000

Table 3: Projected human population 2015-2080 (Carrying Capacity Logistic Growth Model)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **LOGISTICAL GROWTH EXAMPLE OF HUMAN POPULATION (IN THOUSANDS) ON EARTH (YEARS 2015-2080) WITH CARRYING CAPACITY of 10 BILLION PEOPLE** | | | | | | | | |
| YEAR | POPULATION | YEAR | POPULATION | YEAR | POPULATION | YEAR | POPULATION |
| 2015 | 7383009 | 2032 | 7890010 | 2049 | 8321122 | 2066 | 8678899 |
| 2016 | 7414905 | 2033 | 7917469 | 2050 | 8344149 | 2067 | 8697787 |
| 2017 | 7446547 | 2034 | 7944665 | 2051 | 8366922 | 2068 | 8716445 |
| 2018 | 7477934 | 2035 | 7971597 | 2052 | 8389443 | 2069 | 8734874 |
| 2019 | 7509064 | 2036 | 7998264 | 2053 | 8411711 | 2070 | 8753076 |
| 2020 | 7539935 | 2037 | 8024668 | 2054 | 8433729 | 2071 | 8771053 |
| 2021 | 7570549 | 2038 | 8050808 | 2055 | 8455498 | 2072 | 8788807 |
| 2022 | 7600902 | 2039 | 8076686 | 2056 | 8477019 | 2073 | 8806340 |
| 2023 | 7630995 | 2040 | 8102301 | 2057 | 8498294 | 2074 | 8823652 |
| 2024 | 7660826 | 2041 | 8127654 | 2058 | 8519323 | 2075 | 8840747 |
| 2025 | 7690396 | 2042 | 8152745 | 2059 | 8540109 | 2076 | 8857625 |
| 2026 | 7719703 | 2043 | 8177575 | 2060 | 8560651 | 2077 | 8874289 |
| 2027 | 7748747 | 2044 | 8202146 | 2061 | 8580953 | 2078 | 8890740 |
| 2028 | 7777527 | 2045 | 8226456 | 2062 | 8601015 | 2079 | 8906981 |
| 2029 | 7806044 | 2046 | 8250509 | 2063 | 8620840 | 2080 | 8923012 |
| 2030 | 7834297 | 2047 | 8274303 | 2064 | 8640427 |  |  |
| 2031 | 7862285 | 2048 | 8297841 | 2065 | 8659780 |  |  |

Graph 4: Projected human population (2015-2018) logistic growth model

Graph 5: Human population 1950-2080 logistic growth model

**STEP FIVE: ANSWER THE QUESTION(S)**

*Exponential Growth Model*

Based on all factors and variables listed and using the model we formulated, the human population will reach the 21,491,576,000 in the year 2080 based on this model.

*Logistical Growth Model (Carrying Capacity)*

Based on this model, the human population on earth in 2080 will be 8,923,012,000 people.

*Quick Analysis:*

*Exponential Growth Model*

This tells us that over the next 65 years, the population will increase by approximately just over 191.09 percent from the original population. The average growth rate will be held constant at 1.6574% over this 65 year time frame as was the case of the period from 1950-2015. This change in growth rate causes the “j-shape” of the graph curve. As each year passes, the population increases exponentially.

*Logistical Growth Model (Carrying Capacity)*

This tells us that over the next 65 years, the population will increase by approximately just over 20.86 percent from the original population. The average growth rate will be .29% over this 65 year time frame versus an average growth rate of 1.6574% over the period from 1950-2015. This change in growth rate causes the “s-shape” of the graph curve. Simply put, as the model nears the limiting capacity of 10 billion people, growth slows down. This is due to the fact of resource scarcity. As available life giving factors decrease (food, water and land), the population growth will naturally slow at a proportional rate.

*Compare and Contrast*

I have now examined population growth using both an exponential model with no limiting factor and a logistical model that took into account a carrying capacity for the species. The models had some similarities and some large differences.

Similarities: Both models were built upon a previous population. It was not a new data set each year, but rather each model depended on a prior year’s outcome to continue to determine future outcomes. Another way to say this is that both models were dependent on an initial population input. Secondly, both models showed positive growth of the population in every year examined. There has not been any population decline from the year 1950 until 2015 and there is not projected to be any decline in the years 2015-2080 either.

Differences: The biggest difference is the inclusion of a limiting carrying capacity factor in the logistical model where there is not one in the exponential model. The exponential model will account for continued growth for all eternity whereas the logistical model takes into account other factors that detract from continuing growth and will eventually limit the population at a maximum level. The exponential model has a “j-shaped” curve to the data graph where the logistical model has an “s-shaped” curve. The slope continually increases in value in the exponential model (again j shape). In the logistical model, it continually increases for a period then begins to decrease as the graph approaches the asymptote (carrying capacity) until it nears 0 slope. Here that is 10 billion people. When analyzing the actual data below in the exponential model versus the logistical model, there is a projected difference of over 12.5 billion people. This difference is a great example of needed accuracy and great data and modeling to provide a more accurate picture and more importantly, building and selecting the appropriate model for the study.

There are limiting factors to our societal growth. Deaths of all kinds, limited food, water, and shelter/area resources, diseases, etc. all play into our population in a negative capacity, thereby limiting our growth rate until we will ultimately reach a point where the Earth cannot support more human beings. The only way to change this would be to find other habitable planets or spaces (underwater living, air cities, etc.). Only when you make changes to the limiting factors can you expect to be able to move past a limited society capacity. If the world population had no limiting factor, the graph and model we would use would be the exponential growth model. However, we know through other studies and historical data that the logistical growth model is much more accurate to what occurs in real life.

*Behavior*

In my opinion, the most useful part of determining a solid model in any mathematical field is that you can then apply the model with confidence to predict the outcome of various changes. It then becomes a tool rather than just mathematics to further enhance studies, often times in additional fields and areas. For instance, we know between the two models that the exponential model is much more susceptible in regards to a final outcome with a larger initial growth rate than is the logistical model. If we followed an exponential growth rate based on the growth rate 9.3% of 1950-1955, the Earth would have held 575 billion people in 2015 instead of 7.5 billion.

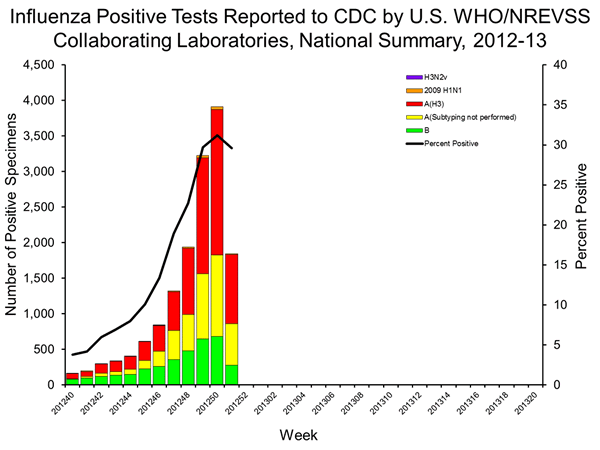
If I increased the carrying capacity in the logistic growth model, the population growth rate would be much larger in the year 2080 as it is projected to be in my model. In fact, when I run the calculations, a carrying capacity of 20 billion would lead to a projected population of 12,642,975,000 versus my projection of 8,923,012,000 people (increased projection of 3.7 billion).

Overall, I found it very valuable to examine both models and compare to actual data to determine which model more correctly fits the actual data. This supporting evidence only strengthens the soundness of the logistical model for growth as it is actually what is occurring in nature. Thus, we can use this model with confidence based on current circumstances/limitations. Only when certain factors change will we have to look deeper at the model.

*Analysis of the Models and Model Selection* It is quite evident after looking at the two separate models for population growth (both exponential and logistical), that the logistical approach and model fits better with what we know in the current human population progression. However, I can also argue that in different circumstances, the exponential growth model would be more effective and accurate.

Looking at the exponential growth model, I argue that it would be a better fit in certain situations. Take a new bacteria for example. A new “superbug” is always of concern to scientists and health organizations. The main reason for this is the fear that the bacteria/virus/superbug will grow exponentially and not provide us time to stop it before it overwhelms us. As I have demonstrated in earlier modules, the exponential growth curve gets steeper with every iteration, so the reproduction speeds up at an accelerated pace. Due to the lack of mitigating factors that could impact, a logistical growth model would not accurately capture the reproduction of said “germ”. The graph below (figure 2) for the first 11 weeks shows just how quickly a germ (in this case influenza) can spread, until they got a handle on things in week 12. As we have seen, however, this is not a good model for when things have limitations like whale or human population growth.

Figure 2:



However, we know that human beings are not viruses, etc. and we have longer reproduction rates with many factors that contribute to the carrying capacity of the species. Viruses and illnesses are just one. Any form of death or limitation to a vital facet of human life (food, water, shelter, habitable space, etc.) puts a limit on how big our species can get under current circumstances. Therefore the model used must account for these limiting factors. Based on our analysis of stability, equilibrium points, vector plots, actual data, etc., we know this model to be the most accurate to reflect the current state of human population growth. We can see the real world data (figure 1) and how this model mimics what is actually happening in reality. You can see the tell-tale s-shape of data with a carrying capacity asymptote. Again, this would not be the best choice of model to study disease outbreak scenarios.

The one thing I would warn against when using the model is complacency. We must constantly review and analyze the variables, initial data, and account for any new changes to assumptions and values. For instance, physical adaptations and/or disorders could drastically affect the model. As a society, our carrying capacity would increase because our constraints would be lessened. However if we didn’t constantly analyze the model, it would lose accuracy.

*Final Recap / Synopsis*

I posed a simple question in this paper. I want to know what the estimated human population on Earth will be in the year 2080. I used several tools, data points, and models to examine this question. I took a look at historical data from the UN world population reports from 1950-2015 (65 year period) to determine figures used to build two models to project what the population will be in the year 2080. I built both a fixed percentage growth (exponential) model and a carrying capacity (logistic) model. Through the use of Excel spreadsheets, graphs, and real world examples/data, I determined that the fixed percentage exponential model vastly overestimates what the population will be in 2080 if all inputs and variables remain as they are today. However, the carrying capacity model provides a much more accurate assessment to estimate the human population over the next 65 year period. Therefore, I selected that model to project the human population to be approximately 8.92 billion people in the year 2080. I hope to live long enough to determine if my projection is correct. Only time will tell!

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